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D-branes as coherent states in the open string channel

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Abstract. We show that bosonic D-brane states may be represented as coherent states in an open string representation. By using the thermo field dynamics (TFD) formalism, we may construct a condensed state of open string modes that encodes the information on the D-brane configuration.
We also introduce a construction alternative to TFD, which does not require one to assume thermal equilibrium. It is shown that the dynamics of the system combined with the geometric properties of the duplication rules of TFD is sufficient to obtain the thermal states and their analytic continuations in a geometric fashion. We use this approach to show that a bosonic D-brane state in the open string sector may also be built as boundary states in a special sense.
Some implications of this study for the interpretation of the open/closed duality and on the kinematic/algebraic structure of an open string field theory are also commented on.

1 Introduction

D-brane states may be constructed as boundary states in the closed string Hilbert space by using the so-called world-sheet duality [1–5]; however, since one may define D-branes as the surfaces where the *open strings* end, it is natural to ask for this definition also in the open string description. In this sense, it has been shown that some exact classical solutions of vacuum string field theory, which is a simplification of Witten's open string field theory, represent D-branes [6]. Other approaches addressed to define boundary states in the open channel were recently proposed [7, 8]. Apart from this, the possibility of describing D-branes in terms of open string states, in contrast to the traditional approaches (in the closed string channel) is specially motivating, since it might shed some light on the open/closed duality, intimately related to the AdS/CFT conjecture.

On the other hand, by virtue of the microscopical description of the black hole entropy [9], one of the most interesting problems concerning D-branes is the development of a model in which their thermodynamical properties and microscopical structure be clarified. The framework presented in this article is devoted to these two purposes simultaneously.

The problem with defining D-brane states in the open string channel is that the boundary conditions are imposed on the operators of the theory rather than on particular states, which could be interpreted as D-brane states in an open string Fock space. However, we claim that this difficult may be solved in the context of the so-called thermo field dynamics (TFD), developed by Takahashi and Umezawa [10–16], where an identical but fictitious copy of the system is introduced. In this framework, variables and degrees of freedom are duplicated as is the original Hilbert space state; then it seems to be possible to construct a state (or a family of states) to describe an open string + brane system.

Thermo field dynamics is a real time approach to quantum field theory at finite temperature [17, 18]. In this formalism one canonically quantizes the fields as operators on a *thermal* Hilbert space and the statistical average of an operator Q is defined as its expectation value in a thermal vacuum state:

$$\frac{\mathrm{Tr}\left[Q\mathrm{e}^{-\beta H}\right]}{Z} = \langle 0(\beta)|Q|0(\beta)\rangle \,. \tag{1}$$

The Hamiltonian evolution of these thermal fields is given by the operator $H-\tilde{H}$, where H and $\tilde{H}$ denote the Hamiltonian of the original system and its non-physical copy respectively. So the fundamental state encoding the statistical information may be represented as follows:

$$|0(\beta)\rangle\rangle = Z^{-1/2}\sum_n \mathrm{e}^{-\beta E_n/2}|n\rangle|\tilde{n}\rangle \,, \tag{2}$$

where $|n;\tilde{n}\rangle$ denotes the nth energy eigenvalue of the two systems.

The idea of using TFD to study D-branes at a finite temperature came up in [19–24], where thermal boundary states are constructed in the closed string channel by considering string coordinates as thermal fields. In contrast, one may compute the free energy of the open string and obtain the self-energy of the D-brane by using the finite temperature dualities [25]. However, this method does

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not provide us with the representation of the D-brane as a (thermal) state in the open string channel. The general goal of this work is precisely to obtain such D-brane states at arbitrary finite temperature in general.

The paper is organized as follows. In Sect. 2 we briefly describe the TFD formalism and propose that a Dp-brane state may be built as the fundamental one in a thermal Hilbert space of open strings. We show this using the standard representation of boundary states in Sect. 2.1. In Sect. 3 we develop a geometric approach in order to formulate D-branes as boundary states in the open string channel. Finally, in Sect. 4 we present the conclusions of this approach and discuss the main consequences and perspectives.

2 Thermo field dynamics and D-brane states

Let us consider the thermodynamics of an open string, that is, let us consider a bosonic open string in contact with a thermal reservoir at temperature β^{-1}. The partition function in the canonical ensemble is

$$Z_{\rm o}(\beta) = \operatorname{tr} \mathrm{e}^{-\beta H_{\rm o}} = \int DX \mathrm{e}^{-S_{W_{\rm o}}[X]} , \qquad (3)$$

where the Euclidean, two-dimensional world-sheet manifold is $W_{\rm o} \sim S^1_\beta \times [0,\pi]$ (β denotes the circle length), where the path integral is realized by summing over histories of the open string fields $X : W_{\rm o} \to M$ satisfying specific boundary conditions that encode the information on the Dp-brane. Since $\partial W_{\rm o} = S^1_{\beta,-} \cup S^1_{\beta,+}$, where the circles $S^1_{\beta,\pm}$ correspond to the points 0 and π respectively and the corresponding boundary conditions are

$$X^i|_{\sigma=0,\pi} = x^i_\pm , \qquad i = p+1, \dots, 25 , \qquad (4)$$
$$\partial_\sigma X^a|_{\sigma=0,\pi} = 0 , \qquad a = 0, 1, \dots, p . \qquad (5)$$

For simplicity, we here assume that both string endpoints belong to the same Dp-brane. If one assumes that an open string interchanges energy-momentum only through its endpoints, then the reservoir should be thought of as to have been placed in the same region that these two point are confined to. This is precisely the Dp-brane surface. So a priori, one could be tempted to identify the Dp-brane with the macroscopical system that acts as the thermal reservoir in itself.

Let us observe that the density matrix $\rho = Z^{-1}\mathrm{e}^{-\beta H_{\rm o}}$ describes the thermal (mixed) state the bosonic open string attached to the Dp-brane in equilibrium, and it encodes the full information on this system in the open channel. Therefore, by virtue of the TFD approach, the thermal state given by this density matrix ρ is *equivalent* to a pure state (the thermal vacuum) in the tensor product of two copies of the quantum Hilbert space of the open string, which entails the quantum and thermal information of the D-brane. In this way, we have a straightforward procedure to construct a representation of a D-brane as a well defined coherent state directly in the open channel, and we do not need to use the standard world-sheet transformation [1–5] to construct the boundary states in the closed picture. For completeness, however, in Sect. 2.1 we will show that in fact this thermal vacuum corresponds to a boundary state via the world-sheet correspondence.

As mentioned before, the TFD algorithm consists first of duplicating the degrees of freedom of the system. To this end a copy of the original Hilbert space may be constructed with a set of creation/annihilation operators that have the same commutation properties as the original ones. The total Hilbert space is the tensor product of the two spaces, $\mathcal{H}_{\rm o} \otimes \widetilde{\mathcal{H}}_{\rm o}$, where in this case $\mathcal{H}_{\rm o}$ denotes the physical quantum states space of the bosonic open string whose endpoints are in contact with the Dp-brane described by (4) and (5).

From now on let us adopt the light cone gauge and use the capital indices $I; J; K; \ldots = 1, \ldots, D-1$ to denote the physical components ($\mu = I, +, -$) of the string embedding. The general solution for the open string coordinates satisfying these boundary conditions reads

$$X^i(t,\sigma) = x^i_-(\pi-\sigma)/\pi + x^i_+(\sigma/\pi) - \sqrt{2\alpha'} \sum_{n\neq 0} \left(\frac{1}{n} \alpha^i_n \mathrm{e}^{-\mathrm{i}nt} \sin(n\sigma) \right) , \qquad (6)$$

$$X^a(t,\sigma) = x^a + 2\alpha' p^a t + 2\mathrm{i}\alpha' \sum_{n\neq 0} \left(\frac{1}{n} \alpha^a_n \mathrm{e}^{-\mathrm{i}nt} \cos(n\sigma) \right) . \qquad (7)$$

The solution corresponding to the non-physical string $\tilde{X}(\tilde{t},\tilde{\sigma})$, with the same boundary conditions, may also be expanded in this basis of solutions. Since introducing finite temperature breaks the Lorentz invariance, we consider this solution in the zero-momentum frame: $p^a = \tilde{p}^a = 0$. Next, these fields are canonically quantized, and, according to the TFD rules [26] the operators of the two systems are built commuting among themselves, so the doubled system is described by two independent strings defining two world sheets.

The Fourier modes may be redefined as

$$a^I_n = \frac{\alpha^I_n}{\sqrt{n}} , \qquad a^{\dagger I}_n = \frac{\alpha^I_{-n}}{\sqrt{n}} , \qquad (8)$$

as the tilde oscillators are, in order to satisfy the extended algebra,

$$\left[a^I_n, a^{\dagger J}_m\right] = \left[\tilde{a}^I_n, \tilde{a}^{\dagger J}_m\right] = \delta_{n,m}\delta^{I,J} ,$$
$$\left[a^{\dagger I}_n, \tilde{a}^J_m\right] = \left[a^{\dagger I}_n, \tilde{a}^{\dagger J}_m\right] = \left[a^I_n, \tilde{a}^J_m\right] = \left[a^I_n, \tilde{a}^{\dagger J}_m\right] = 0 . \qquad (9)$$

The standard vacuum in this extended theory is defined by

$$a^I_n|0\rangle\rangle = \tilde{a}^I_n|0\rangle\rangle = 0 , \qquad (10)$$

for $n > 0$ and $|0\rangle\rangle = |0\rangle \otimes |\widetilde{0}\rangle$ as usual. However, the physical thermal fundamental state shall be obtained from this through a Bogoliubov transformation, $\mathrm{e}^{-\mathrm{i}G}$, which entangles the states of the two independent Hilbert spaces. This is given by the following relation:

$$|0(\theta)\rangle = \mathrm{e}^{-\mathrm{i}G}|0\rangle\rangle = \prod_{n=1}\left[\left(\frac{1}{\cosh(\theta_n)}\right)^{D-2}\mathrm{e}^{\tanh(\theta_n)\delta_{IJ}a_n^{\dagger I}\tilde{a}_n^{\dagger J}}\right]|0\rangle\rangle\,. \tag{11}$$

Here θ denotes the set of transformation parameters. By applying the operator (4) to the state $|0\rangle\rangle$ one may see that this state encloses the specific values of the endpoints' positions, and furthermore this is not modified by the Bogoliubov transformation.

The thermal creation and annihilation are also transformed according to

$$a_n^I(\theta_n) = \mathrm{e}^{-\mathrm{i}G}a_n^I\mathrm{e}^{\mathrm{i}G} = \cosh(\theta_n)a_n^I - \sinh(\theta_n)\tilde{a}_n^{\dagger I}\,, \tag{12}$$
$$\tilde{a}_n^I(\theta_n) = \mathrm{e}^{-\mathrm{i}G}\tilde{a}_n^I\mathrm{e}^{\mathrm{i}G} = \cosh(\theta_n)a_n^I - \sinh(\theta_n)\tilde{a}_n^{\dagger I}\,. \tag{13}$$

As the Bogoliubov transformation is canonical, the thermal operators obey the same commutation (9). These operators annihilate the state written in (11) defining it as the vacuum. By using the Bogoliubov transformation, the relations

$$a_n^I(\theta_n)|0(\theta)\rangle = \tilde{a}_n^I(\theta_n)|0(\theta)\rangle = 0 \tag{14}$$

give rise to the so-called thermal state conditions:

$$\left[a_n^I - \tanh(\theta_n)\tilde{a}_n^{\dagger I}\right]|0(\theta)\rangle = 0\,, \tag{15}$$
$$\left[\tilde{a}_n^I - \tanh(\theta_n)a_n^{\dagger I}\right]|0(\theta)\rangle = 0\,. \tag{16}$$

Next the physical open string Fock space is constructed by applying the thermal creation operators to the vacuum (11) that may consistently be identified with the D-brane state. In fact, if the modes of the open string attached to the D-brane are created from this state, precisely in the absence of such string excitations, the D-brane on its own must be associated to the fundamental state.

Finally, the thermal open string vacuum is completely defined by minimizing the free energy

$$F = U - \frac{1}{\beta}S\,, \tag{17}$$

with respect to the transformation's parameters θ [10, 11]. Here U is given by computing the matrix elements of the open string Hamiltonian in the thermal vacuum and S is the expectation value of the entropy operator $K \equiv -\sum_{n=1} N_n \ln N_n$ in this state. The number operator is defined by

$$N_n = a_n^{\dagger I}a_n^J\delta_{IJ}\,, \tag{18}$$

whose expectation value is proportional to $\sinh^2\theta_n$; therefore we get

$$K = -\sum_{n=1}\left\{a_n^{\dagger I}a_n^J\delta_{IJ}\ln\left(\sinh^2(\theta_n)\right) - a_n^I a_n^{\dagger J}\delta_{IJ}\ln\left(\cosh^2(\theta_n)\right)\right\}. \tag{19}$$

Therefore, the solution for the angular parameters θ_n is given by the Bose–Einstein distribution:

$$\sinh^2\theta_n = (\mathrm{e}^{\beta E_n} - 1)^{-1}\,. \tag{20}$$

Notice that, although expressed in the open sector, the thermal state conditions (15) and (16) together with the thermal equilibrium requirement (in order to fix the free parameters θ_n) determine the state of the system, so that a D-brane state is built in the closed channel. So, in this sense we could see (15) and (16) as a sort of boundary state condition in the open channel. In Sect. 3 we will construct a geometrical approach to the boundary states in the open channel, where this interpretation arises explicitly.

It is easy to see that thermal states are not eigenstates of the original Hamiltonian, but they are eigenstates of the combination

$$\widehat{H} = H - \tilde{H}\,, \tag{21}$$

in such a way that $\widehat{H}$ plays the rôle of the Hamiltonian, generating temporal translation in the thermal Fock space. Let us point out that the physical variables are described by the non-tilde operators. This is then the Hamiltonian that governs the dynamical evolution of the D-brane and its excitations, which correspond to an open string attached to it.

The state (11) describes a condensate of entangled open string modes localized on the D-brane surface. Since this is a coherent state, it constitutes a macroscopical object [14] that may be identified with the D-brane. So we conclude this part by emphasizing that (11) describes the microscopical structure of the D-brane in terms of open string modes.

Analytical continuation. Let us remark on analytical continuation that in spite of D-brane states having been constructed here at arbitrary finite temperature, the parameter β can be analytically continued to the complex plane and in particular to purely imaginary values, $\beta = \mathrm{i}\lambda$, $\lambda \in \mathbb{R}$ [27], in order to describe states without temperature. In this context, however, the quantity τ should not be interpreted as a time evolution parameter in order to describe stationary states, but, as clarified in the construction of Sect. 3, it could be seen as a sort of "time delay" between the physical system and its copy.

2.1 D-brane states from the current closed string description

We found a way to represent a D-brane as a state in the open channel, and we do not need to transform the problem by going to the closed representation; however, by using world-sheet duality [1–5], one may verify that the thermal vacuum (11) consistently *corresponds* to a current boundary state in the closed channel.

Let us first motivate our proposal in the context of this duality. In fact, the interaction between two D-branes, or, according to the case analyzed in this paper, the self-interaction of only one D-brane. is given by the vacuum fluctuations of an open string ending on them and propagating in a loop with periodic Euclidean time $t \in [0, \beta]$ (the Casimir effect). Graphically, the topology of this open string world sheet is a cylinder ending on the two branes. Since the theory is conformally invariant, one can find a conformal transformation such that the world-sheet coordinates are exchanged and the cylinder corresponds to the tree diagram of a boundary state of a closed string being created in one of the D-branes, propagating for a while, and annihilated on the other brane. These boundary states are identified with the D-brane states in the closed string channel. The crucial observation of our work is that one may avoid the world-sheet transformation in this algorithm and recognize the D-brane states in the open sector. The second remarkable point in this observation is that, roughly speaking, the one-loop cylinder diagram in fact characterizes a thermal state by virtue of the Euclidean period. Let us now show this in detail.

Consider a particular initial configuration of a closed string,

$$\partial_t X^a|_{t=0} = 0\,, \qquad a = 0, 1, \ldots, p\,, \tag{22}$$

$$X^i|_{t=0} = x^i\,, \qquad i = p+1, \ldots, 25\,. \tag{23}$$

Because the operators $\partial_t X^a$ and X^i commutate among themselves at the same time, a specific configuration of these constitutes a definite *state* in the quantum Hilbert space of a closed string. This is called a boundary state, which is interpreted as the Dp-brane state in itself. It may be expressed as a coherent state of closed string modes and its form is remarkably similar to (11) [1–5].

The transition amplitude from this state, corresponding to the initial configuration (22) and (23) and denoted by $|B_p(t=0)\rangle$, into a final one $|B_p(t=-\mathrm{i}\pi)\rangle$ (defined by conditions similar to (22) and (23)) through an imaginary time interval $-\mathrm{i}\pi$, is given by

$$\langle B_p(t=-\mathrm{i}\pi)|\mathrm{e}^{-\pi H_\mathrm{c}}|B_p(t=0)\rangle = \int_\Gamma DX \mathrm{e}^{-S_{W_\mathrm{c}}[X]}\,, \tag{24}$$

where H_c is the closed string Hamiltonian. This may be represented as a sum over histories as expressed in the right hand side of this identity, where $W_\mathrm{c} \sim S^1_\beta \times [0, \pi]$ is the closed world-sheet topology whose boundary are two circles, which we denote by $S^1_{\beta,\pm}$. Then Γ represents the set of histories of one closed string ending on these two circles, whose states are fixed by the configurations (22) and (23).

Then by considering the world-sheet transformation $\sigma, t \to t, \sigma$, W_c transforms into W_o, and the above sum over histories coincides with (3). Finally, one straightforwardly obtains the state (11) according to the procedure previously shown, which encodes the information of the boundary state. In fact, it is clear that the state $|0(\beta)\rangle$ and the boundary state $|B_p\rangle$ may both be used to calculate the significant observables/amplitudes in the respective representations, and they are corresponding in the proper sense.[1] If one inserts any operator Q as in (1), the quantity $\langle 0(\beta)|Q|0(\beta)\rangle$ must be identified with the amplitude

$$A_{B_p}[Q_\mathrm{c}] = \int_{\Gamma[B_p]} DX Q_\mathrm{c} \mathrm{e}^{-S_{W_\mathrm{c}}[X]}\,, \tag{25}$$

computed in the closed channel. This integral is a sum over all world-sheet embeddings subject to the boundary conditions (22) and (23), as discussed above. In particular, if we take Q to be a product of operators $\{X(\sigma_i, t_i), i = 1, \ldots, n\}$ valued on a collection of n different world-sheet points, we get the n-point thermal Green function of the open string – the propagators in the thermal vacuum. On the other hand, Q_c corresponds to the same object in the closed channel, the n-point correlation function for n points of the closed string, which are defined by exchange of the world-sheet coordinates: $(\sigma_i, t_i)_\mathrm{open} \to (t_i, \sigma_i)_\mathrm{closed}$, according to the discussion above. This prescription may be extended: we then consider products of different derivatives of these operators. This completes the argument on our initial statement.

3 Geometric formulation and boundary states in the open string representation

The goal of this section is to show that D-brane states may be constructed as boundary states even in the open sector in an appropriate sense. In other words, the boundary state conditions whose solution is $|B_\mathrm{open}\rangle$ may be imposed also in the open string channel as conditions on states rather than operators in a way similar to the Gupta–Bleuler standard procedure. In fact, these conditions consist of fixing of non-physical variables in terms of physical ones[2] (on their respective spatial boundaries), which is analogous to a gauge fixing. So in this study, we rigorously refer to a boundary state in the context of open strings in this precise sense; however, we would like to emphasize here that in spite of these states maybe being fixed initially[3] and, as argued before, as they carry the same information as the closed string boundary states, it should be clearly differentiated from the concept of a "boundary state in the open string channel" properly introduced in [7, 8], where such states may actually describe the emission and absorption of the (open) strings by D-branes as the usual boundary state does for the closed strings.

To do this we introduce a purely geometrical approach that only requires the duplication structure of TFD and in which the thermodynamic concepts may be ignored. The dynamical information of the system is sufficient to determine these states. This technique is interesting in itself, because it seems to be applicable to other situations with boundary conditions (see e.g. [27]); it is similar to TFD but incorporates some new ingredients related with the dynamical properties of the system and with its geometry.

[1] Transition amplitudes between different D-brane states will be considered in a forthcoming paper.

[2] These may alternatively be interpreted as a selection of physical states that in this case shall describe the D-brane and its excitation (open) modes at finite temperature.

[3] Then they are preserved by the evolution of the system, generated by the total Hamiltonian (21).

Note that the main idea underlying the TFD approach to this problem (emphasized in this new construction) is the possibility of describing the contact of one open string with a D-brane by effectively substituting such an object, whose dynamics and degrees of freedom are unknown in principle, by another fictitious string. In fact, it seems to be natural to think that the effective degrees of freedom of the brane, which are activated by the energy-momentum exchange with the physical string, be in correspondence with a single-string degrees of freedom. This is what we call the fictitious string or "hole" (as used in the TFD literature).

In geometric terms, we may represent a string ending on the D-brane surface, while the fictitious string lives on the other side [28] and ends on the *same* brane as required by the TFD duplication rules. The invariance group of the string attached to a Dp-brane is $G_p \equiv \mathrm{SO}(1,p) \times \mathrm{SO}(D-p)$, and another equal symmetry $\tilde{G}_p$ should be attributed to the fictitious string variables.

The boundary conditions to quantize the open string are

$$X^i|_{\sigma=0,\pi} = x^i\,, \qquad i = p+1,\ldots,25\,, \tag{26}$$

$$\partial_\sigma X^a|_{\sigma=0,\pi} = 0\,, \qquad a = 0,1,\ldots,p\,, \tag{27}$$

and they must be the same for $\tilde{X}$, in order to have a copy of the original system as required by TFD:

$$\tilde{X}^i|_{\tilde{\sigma}=0,\pi} = x^i\,, \qquad i = p+1,\ldots,25\,, \tag{28}$$

$$\partial_\sigma \tilde{X}^a|_{\tilde{\sigma}=0,\pi} = 0\,, \qquad a = 0,1,\ldots,p\,, \tag{29}$$

which defines another string in contact with a Dp-brane in the same position x^i. Once more we assume the Hilbert space $\mathcal{H}_\mathrm{o} \otimes \tilde{\mathcal{H}}_\mathrm{o}$. By using a part of the symmetry $\tilde{G}_p$, we may translate its endpoint along the Dp-brane hyperplane to coincide with those of the physical string. So we may see this procedure as a sort of gauge fixing, which, according to the Gupta–Bleuler prescription, shall be imposed on the states as follows:

$$\tilde{X}^\mu(\tilde{\sigma},\tilde{t})|_{\tilde{\sigma}=0,\pi} - X^\mu(\sigma,t)|_{\sigma=0,\pi}|B_\mathrm{open}\rangle = 0\,, \tag{30}$$

whose components $\mu = i = p+1,\ldots,25$ are trivially satisfied due to (26) and (28); furthermore, this manifestly implies that the respective right hand sides of these two conditions must coincide. On the other hand, in order to ensure the smooth gluing of both open strings in their respective endpoints, we shall require continuity of the first derivative with respect to the respective strings parameters in their boundaries. This condition is

$$\partial_{\tilde{\sigma}}\tilde{X}^\mu(\tilde{\sigma},\tilde{t})|_{\tilde{\sigma}=0,\pi} - \partial_\sigma X^\mu(\sigma,t)|_{\sigma=0,\pi}|B_\mathrm{open}\rangle = 0\,, \tag{31}$$

whose components $\mu = a = 0,\ldots,p+1$ trivially annihilate all the states of the Hilbert space by virtue of (27) and (29). Then (30) and (31) constitute D non-trivial conditions on the D-brane states $|B_\mathrm{open}\rangle$ ($D-2$, considering only the physical components in the light cone gauge).

Let us remark that the tilde variables were fixed in terms of the non-tilde ones by these conditions, which clearly break the symmetry $G_p \times \tilde{G}_p$ into the original group G_p.

The map between the tilde and non-tilde operators is defined by the following tilde (or dual) conjugation rules [26]:

$$\begin{aligned}
(XY)^\sim &= \tilde{X}\tilde{Y}\,,\\
(cX+Y)^\sim &= c^*\tilde{X}_i + \tilde{Y}_j\,,\\
(X^\dagger)^\sim &= (\tilde{X})^\dagger\,,\\
(\tilde{X})^\sim &= X\,,\\
[\tilde{X},Y] &= 0\,.
\end{aligned} \tag{32}$$

Considering the solutions (6) and (7) in the light cone gauge, one may use these rules to construct the fictitious copy of the string $\tilde{X}^I$ defined on an independent [28] worldsheet manifold whose coordinates are $\tilde{t},\tilde{\sigma}$, and we get

$$\begin{aligned}\tilde{X}^i(\tilde{t},\tilde{\sigma}) &= x^i_-(\pi-\tilde{\sigma})/\pi + x^i_+(\tilde{\sigma}/\pi)\\
&\quad - \sqrt{2\alpha'}\sum_{n\neq 0}\left(\frac{1}{n}\tilde{\alpha}^i_n \mathrm{e}^{\mathrm{i}n\tilde{t}}\sin(n\tilde{\sigma})\right),\end{aligned} \tag{33}$$

$$\tilde{X}^a(\tilde{t},\tilde{\sigma}) = \tilde{x}^a + 2\alpha'\tilde{p}^a\tilde{t} - 2\mathrm{i}\alpha'\sum_{n\neq 0}\left(\frac{1}{n}\tilde{\alpha}^a_n \mathrm{e}^{\mathrm{i}n\tilde{t}}\cos(n\tilde{\sigma})\right). \tag{34}$$

If we finally redefine the mode number $n \to -n$[4] in each term of this expression, the solution reads

$$\begin{aligned}\tilde{X}^i(\tilde{t},\tilde{\sigma}) &= x^i_-(\pi-\tilde{\sigma})/\pi + x^i_+(\tilde{\sigma}/\pi)\\
&\quad - \sqrt{2\alpha'}\sum_{n\neq 0}\left(\frac{1}{n}\tilde{\alpha}^i_{-n} \mathrm{e}^{-\mathrm{i}n\tilde{t}}\sin(n\tilde{\sigma})\right),\end{aligned} \tag{35}$$

$$\tilde{X}^a(\tilde{t},\tilde{\sigma}) = \tilde{x}^a + 2\alpha'\tilde{p}^a\tilde{t} + 2\mathrm{i}\alpha'\sum_{n\neq 0}\left(\frac{1}{n}\tilde{\alpha}^a_{-n} \mathrm{e}^{-\mathrm{i}n\tilde{t}}\cos(n\tilde{\sigma})\right). \tag{36}$$

Although the time evolution of both strings is respectively given by the independent time parameters t and $\tilde{t}$, they both shall parameterize the same time direction (in the target) in order to preserve the gauge choice; then in particular we may define them up to a general shift $\tilde{t} \equiv t+\tau$. In the light cone frame both time parameters are given by the coordinate X^+, and they only can be related up to an additive number. Below, we will discuss the important meaning of this time delay between both strings.

An appropriate initial choice of the non-physical/physical gluing of the variables in the boundary of the system according to the conditions (30) and (31) shall in fact determine boundary states; then these states shall be built such that these conditions be satisfied for all t. By requiring this in the expansion in the modes of (30) and (31), we have the results

$$\tilde{a}^I_n - \mathrm{e}^{\mathrm{i}n\tau} a^{\dagger I}_n |B_\mathrm{open}\rangle = 0\,, \tag{37}$$

$$x^a - \tilde{x}^a + 2\alpha'\tilde{p}^a\tau|B_\mathrm{open}\rangle = 0\,, \tag{38}$$

$$\tilde{p}^a - p^a|B_\mathrm{open}\rangle = 0\,, \tag{39}$$

[4] Notice that this change is related to the structure of the Hamiltonian (21) for the duplicated system.

where we have used (8) to define the canonical creation/annihilation operators. Then the solution of these equations may be expressed as

$$|B_{\text{open}}\rangle = N'\delta(x^a - \tilde{x}^a + 2\alpha'\tilde{p}^a\tau)\delta(\tilde{p}^a - p^a) \prod_{I,n>0} e^{q_n a_n^{\dagger I} \tilde{a}_n^{\dagger I}} |0\rangle\rangle\,, \tag{40}$$

where $q_n = e^{in\tau}$, $N' \equiv N\delta(X^i|_{\sigma=0,\pi} - x^i_\pm)\delta(\tilde{X}^i|_{\tilde{\sigma}=0,\pi} - x^i_\pm)$ and N is the normalization constant.[5]

Now we observe the remarkable fact that the time shift τ may be an arbitrary complex number. In particular, if this is taken to be a purely imaginary number $\tau \equiv -i\beta/2$, one may define the physical time t parameter to be real, and, in this case, the evolution of the fictitious system will be parameterized precisely by the real part of $\tilde{t}$ (where $\tau \equiv \mathrm{Im}(\tilde{t})$).

If $\tau \equiv -i\beta/2$, and if one assumes reality conditions also for the fictitious variables, (38) splits into

$$x^a - \tilde{x}^a|B_{\text{open}}\rangle = 0\,, \tag{41}$$

$$\tilde{p}^a|B_{\text{open}}\rangle = 0\,. \tag{42}$$

Thus the solution of (37), (39), (41) and (42) may be written as

$$|B_{\text{open}}\rangle = N'\delta(x^a - \tilde{x}^a)\delta(\tilde{p}^a)\delta(p^a) \prod_{I,n>0} e^{q_n a_n^{\dagger I} \tilde{a}_n^{\dagger I}} |0\rangle\rangle\,. \tag{43}$$

In this case $q_n = e^{-n\beta/2}$. Defining $\theta_n = \tanh^{-1} q_n(\tau)$, the string mode occupation number is given by $N_n = \sinh^2\theta_n$, which agrees with the Bose–Einstein distribution of string modes at the temperature β^{-1}, and this state remarkably coincides with (11) found in Sect. 2. Therefore, we showed here that the effect *produced* by the brane on the open string at finite temperature is equivalent to the effect due to another string, joined to the first one through their boundaries, but forwarded by an imaginary time interval (which in this formalism need not to be compactified to the circle as usual; in particular, in the imaginary time approach [29–31]). Furthermore, this time delay may be interpreted in terms of the temperature of the brane.

As pointed out in Sect. 2, one may consider the analytical continuation of the parameter $\tilde{\tau}$ to take real values and the boundary states are given by (40). Although this does not arise from the traditional TFD construction, we may observe here that this number should not be interpreted as a time evolution parameter (as often it is believed to be) but as a relative delay.

Finally, it is interesting to remark that this may be seen as the geometrical representation of the postulate of it being non-physical for the tilde system, since the "world manifold" associated with it is causally disconnected of the parallel world manifold of the real system due to the imaginary time delay between them. However, there is quantum entanglement between them, which gives room for thermal states. In contrast if $\tau \in \mathbb{R}$, in principle both systems may superpose and this may be a cause of interference among them. This may describe dynamical (out of the thermal equilibrium) effects of the brane. We furthermore believe that this also may open the possibility of studying interactions at finite temperature [32].

These are some of the new remarks and perspectives emphasized by this formulation and these shall be studied in detail elsewhere.[6]

4 Concluding remarks and outlook

In this work we obtained the open string representation of a bosonic Dp-brane state and its generalization at finite temperature. A remarkable strong feature of this description is that the Dp-branes are emphasized as vacuum states of a open string Fock space. On the other hand, this approach handles a model for Dp-branes, where its macroscopical nature is manifest [14]. Notice in particular that if we consider an ensemble of open strings attached to the same Dp-brane, then in the thermodynamic limit this may be seen as a sort of *medium* extended on p spatial dimensions, filled with open string modes.

Let us observe that the configuration discussed in Sect. 3 constitutes a composite between two open strings (so one may say that the $|B_{\text{open}}\rangle$ represent composite states) and notice, in addition, that from a strictly topological point of view, these two form a closed string. This observation could help to understand what is the manifest correspondence between both open/closed representations of D-brane states in the TFD language, and this furthermore gives rise to deeper question: may closed string states be viewed as bound states of open strings?

In this sense, we believe that the open/closed duality could be properly interpreted as a correspondence between closed string states and *thermal* (mixed) states of open strings. In fact, according to the arguments explained in Sect. 3, if we search for an open string configuration at thermal equilibrium, we generically may read it as a path integral formulation of a closed string theory. On the other hand, let us remark that this fact suggests that the kinematical structure of an open string field theory shall require a $C^\star$ algebra [34, 35], and this is an indication of how the closed string sector could be recovered in such a theory.

In future work we will investigate the thermal stability of the D-brane configurations and the possibility of describing black branes using these ideas. According to [36–38], the infrared behavior of theories whose dual bulk-gravities contain a black brane is governed by hydrodynamics, and the main observation in this sense is the existence of a universal value for the ratio of shear viscosity

[5] Note that the centers of mass of both strings follow the same trajectory but the position of one of them is forwarded by $2\alpha'\tilde{p}^a\tau$ with respect to the other one.

[6] We should mention that the construction presented here and the geometric description of thermal closed string given in [33] have some similar aspects.

to entropy density [39], which should be investigated in the context of an appropriate microscopical model.

Finally, the connection of our results with the closed string description of thermal D-branes [19–24] should be clarified.

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